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Determination of Third- and Fourth-Order Longitudinal Elastic Constants by Shock Compression Techniques—Application to Sapphire and Fused Quartz*

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A number of solids sustain large elastic compressions under shock-wave loading. In these solids, measurements of the stress and compression in the direction of shock propagation can be used to calculate both third- and fourth-order longitudinal elastic constants if measurements are carried out over a wide range of compressions. Only limited measurements of fourth-order constants have been previously determined by other techniques. Determinations of third-order constants under these large elastic compressions afford the opportunity to test the applicability of the finite-strain formulation of constitutive relations. A general method for calculating these third- and fourth-order constants is presented and applied to shock compression data for sapphire and fused quartz. For sapphire, it is found that $C_{111}\approx C_{333}=-(3.3\pm0.3)\times10^4$ kbar and $C_{1111}\approx C_{3333}=+(5.0\pm1.5)\times10^5$ kbar. For fused quartz, it is found that $C_{111}=+(5.5\pm0.1)\times10^3$ kbar and $C_{1111}=+(110\pm10)\times10^3$ kbar. The technique and method of analysis seem generally applicable to solids that exhibit elastic limits of a few percent of their longitudinal elastic constants.

INTRODUCTION

When subjected to shock-wave compression, a number of solids are observed to exhibit unusually large elastic limits. Noting that large elastic compressions can be achieved under shock compression, Fowles[1] expressed the finite-strain high-order elastic constant theory in terms suitable for analysis of elastic shock-compression data. He proposed that longitudinal fourth-order elastic constants could be computed from shock-compression data if the second- and third-order elastic constants were known. From this analysis, the longitudinal fourth-order elastic constants of α quartz were computed from the shock-compression data in the elastic range, i.e., below the Hugoniot elastic limit. The present paper extends the analysis of shock-compression data to the determination of both third- and fourth-order longitudinal elastic constants.

A number of measurements of third-order elastic constants have been accomplished with static compression techniques, including measurements on Ge,[2–6] MgO,[2] Si,[4,6] fused quartz,[2] α quartz,[7] and sapphire.[8] Measurements of these third-order constants are of both fundamental and applied interest. The third-order constants are associated with anharmonicity of a crystal lattice; hence, they may be used to calculate generalized Grüneisen parameters.[9] Furthermore, quantitative descriptions of acoustic amplification at microwave frequencies in solids[10,11] requires knowledge of the third-order elastic constants. If piezoelectric solids are used for amplification, high-order piezoelectric constants are also important.[12–14] The attenuation in microwave delay lines is influenced by the Akhiezer phonon–phonon interaction mechanism,[15] which can be calculated from the third-order elastic constants.

Only a limited number of fourth-order elastic constant measurements have been accomplished and there is no established technique for their determination. In addition to Fowles's measurements, fourth-order constants of several cesium halides have been measured by ultrasonic techniques,[16] and several combinations of fourth-order constants of fused quartz have been determined in uniaxial tension experiments.[17] Fourth-order constants have not yet been required for interpretation of microwave phenomena, but it has been suggested that harmonic generation in stressed crystals could be used to determine fourth-order constants.[18]

Although, at present, only longitudinal elastic constants can be determined from shock-compression measurements, it appears that these longitudinal constants are often of interest. Furthermore, the determination of the third-order constant under large compressions permits a test of the formulation of the finite-strain theory.

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compressions of 0.5%. Between 0.5% and 2.5%, the third-order constant gives a significant contribution, while the fourth-order constant gives a significant contribution for compressions from 2.5% to 6%.

IV. CONCLUSION

Analysis of the sapphire and fused-quartz shock-compression data demonstrates that the analytical method employed permits the determination of both third- and fourth-order longitudinal elastic constants. Although the third-order constants may be determined by other techniques, the fourth-order constants have been determined only by shock-compression techniques. The method is limited to solids that sustain large elastic compressions in uniaxial strain; however, a number of solids exhibit large Hugoniot elastic limits. The materials with known large Hugoniot elastic limits[24] include: sapphire, α quartz, MgO, Ge, Si, CdS, InSb, TiO$_2$, B$_4$C, BeO, and yttrium iron garnet. Thus, the method may be applied to a reasonably large number of solids of technical interest. Although shock-compression measurements have been performed on all these solids, the measurements are usually limited to several discrete stress–volume points, and these data are insufficient for the determination of third- and fourth-order constants.

Even though there is some question as to the appropriateness of extending the theory to compression data to fourth order,[9] it is clear that the experimentally observed compressions of α quartz, sapphire, and fused quartz can be adequately described by the fourth-order constant development. Furthermore, it appears that shock-compression measurements can play a generally useful role in the determination of longitudinal third- and fourth-order elastic constants. If precise stress-versus-volume relations can be obtained under large elastic compressions, it appears that the finite-strain formulations can be given an evaluation. The present measurements are somewhat limited in accuracy, but it appears that the finite-strain formulation given here gives an appropriate description to both the ultrasonic and shock-compression data of sapphire and fused quartz.

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